When the integrand has the form $f(ax + b)$, the substitution $u = ax + b$ is often effective.

SOLUTION

a. Let the new variable be $u = x + 3$ and then $du = dx$. Because we have changed the variable of integration from x to u, the limits of integration must also be expressed in terms of u. In this case,

$$x = 0 \text{ implies } u = 0 + 3 = 3 \quad \text{Lower limit}$$
$$x = 2 \text{ implies } u = 2 + 3 = 5 \quad \text{Upper limit}$$

The entire integration is carried out as follows:

$$\int_0^2 \frac{dx}{(x+3)^3} = \int_3^5 u^{-3}\,du \quad \text{Substitute } u = x + 3,\ du = dx.$$
$$= -\frac{u^{-2}}{2}\bigg|_3^5 \quad \text{Fundamental Theorem}$$
$$= -\frac{1}{2}(5^{-2} - 3^{-2}) = \frac{8}{225} \quad \text{Simplify.}$$

b. Notice that a multiple of the derivative of the denominator appears in the numerator; therefore, we let $u = x^2 + 1$. Then $du = 2x\,dx$, or $x\,dx = \frac{1}{2}du$. Changing limits of integration,

$$x = 0 \text{ implies } u = 0 + 1 = 1 \quad \text{Lower limit}$$
$$x = 4 \text{ implies } u = 4^2 + 1 = 17 \quad \text{Upper limit}$$

Changing variables, we have

$$\int_0^4 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_1^{17} u^{-1}\,du \quad \text{Substitute } u = x^2 + 1,\ du = 2x\,dx.$$
$$= \frac{1}{2}(\ln|u|)\bigg|_1^{17} \quad \text{Fundamental Theorem}$$
$$= \frac{1}{2}(\ln 17 - \ln 1) \quad \text{Simplify.}$$
$$= \frac{1}{2}\ln 17 \approx 1.417. \quad \ln 1 = 0$$

c. Let $u = \sin x$, which implies that $du = \cos x\,dx$. The lower limit of integration becomes $u = 0$ and the upper limit becomes $u = 1$. Changing variables, we have

$$\int_0^{\pi/2} \sin^4 x \cos x\,dx = \int_0^1 u^4\,du \quad u = \sin x,\ du = \cos x\,dx$$
$$= \left(\frac{u^5}{5}\right)\bigg|_0^1 = \frac{1}{5}. \quad \text{Fundamental Theorem}$$

Related Exercises 35–44

The Substitution Rule enables us to find two standard integrals that appear frequently in practice, $\int \sin^2 x\,dx$ and $\int \cos^2 x\,dx$. These integrals are handled using the identities

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1 + \cos 2x}{2}.$$

EXAMPLE 6 Integral of $\cos^2 \theta$ Evaluate $\int_0^{\pi/2} \cos^2 \theta\,d\theta$.

SOLUTION Working with the indefinite integral first, we use the identity for $\cos^2 \theta$:

$$\int \cos^2 \theta\,d\theta = \int \frac{1 + \cos 2\theta}{2}\,d\theta = \frac{1}{2}\int d\theta + \frac{1}{2}\int \cos 2\theta\,d\theta.$$

See Exercise 86 for a generalization of Example 6. Trigonometric integrals involving powers of $\sin x$ and $\cos x$ are explored in greater detail in Section 7.2.

The change of variables $u = 2\theta$ is now used for the second integral, and we have

$$\int \cos^2 \theta\,d\theta = \frac{1}{2}\int d\theta + \frac{1}{2}\int \cos 2\theta\,d\theta$$
$$= \frac{1}{2}\int d\theta + \frac{1}{2}\cdot\frac{1}{2}\int \cos u\,du \quad u = 2\theta,\ du = 2\,d\theta$$
$$= \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + C. \quad \text{Evaluate integrals; } u = 2\theta.$$

Using the Fundamental Theorem of Calculus, the value of the definite integral is

$$\int_0^{\pi/2} \cos^2 \theta\,d\theta = \left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\bigg|_0^{\pi/2}$$
$$= \left(\frac{\pi}{4} + \frac{1}{4}\sin \pi\right) - \left(0 + \frac{1}{4}\sin 0\right) = \frac{\pi}{4}.$$

Related Exercises 45–50

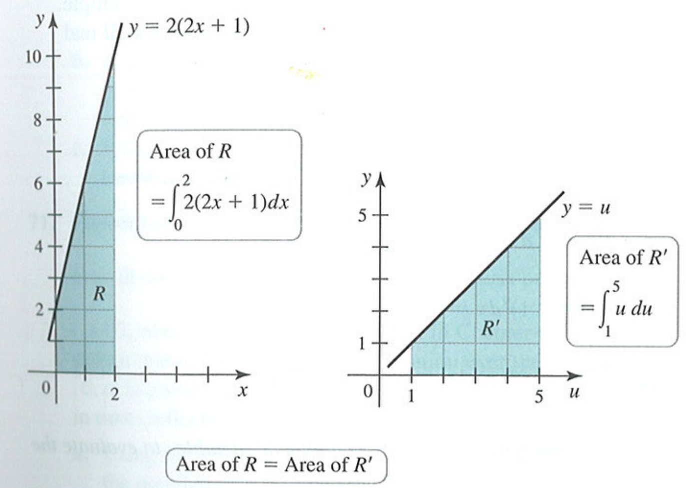


FIGURE 5.57

Geometry of Substitution

The Substitution Rule may be interpreted graphically. To keep matters simple, consider the integral $\int_0^2 2(2x + 1)\,dx$. The graph of the integrand $y = 2(2x + 1)$ on the interval $[0, 2]$ is shown in Figure 5.57, along with the region R whose area is given by the integral. The change of variables $u = 2x + 1$, $du = 2\,dx$, $u(0) = 1$, and $u(2) = 5$ leads to the new integral

$$\int_0^2 2(2x + 1)\,dx = \int_1^5 u\,du.$$

Figure 5.57 also shows the graph of the new integrand $y = u$ on the interval $[1, 5]$ and the region R' whose area is given by the new integral. You can check that the areas of R and R' are equal. An analogous interpretation may be given to more complicated integrands and substitutions.

QUICK CHECK 3 Changes of variables occur frequently in mathematics. For example, suppose you want to solve the equation $x^4 - 13x^2 + 36 = 0$. If you use the substitution $u = x^2$, what is the new equation that must be solved for u? What are the roots of the original equation?

SECTION 5.5 EXERCISES

Review Questions

1. On which derivative rule is the Substitution Rule based?
2. Explain why the Substitution Rule is referred to as a change of variables.
3. The composite function $f(g(x))$ consists of an inner function g and an outer function f. When doing a change of variables, which function is often a likely choice for a new variable u?
4. Find a suitable substitution for evaluating $\int \tan x \sec^2 x\,dx$, and explain your choice.
5. When using a change of variables $u = g(x)$ to evaluate the definite integral $\int_a^b f(g(x))g'(x)\,dx$, how are the limits of integration transformed?
6. If the change of variables $u = x^2 - 4$ is used to evaluate the definite integral $\int_2^4 f(x)\,dx$, what are the new limits of integration?
7. Find $\int \cos^2 x\,dx$.
8. What identity is needed to find $\int \sin^2 x\,dx$?

Basic Skills

9–12. Trial and error *Find an antiderivative of the following functions by trial and error. Check your answer by differentiation.*

9. $f(x) = (x + 1)^{12}$
10. $f(x) = e^{3x+1}$
11. $f(x) = \sqrt{2x + 1}$
12. $f(x) = \cos(2x + 5)$

13–16. Substitution given *Use the given substitution to find the following indefinite integrals. Check your answer by differentiation.*

13. $\int 2x(x^2 + 1)^4\, dx,\ u = x^2 + 1$

14. $\int 8x \cos(4x^2 + 3)\, dx,\ u = 4x^2 + 3$

15. $\int \sin^3 x \cos x\, dx,\ u = \sin x$

16. $\int (6x + 1)\sqrt{3x^2 + x}\, dx,\ u = 3x^2 + x$

17–28. Indefinite integrals *Use a change of variables to find the following indefinite integrals. Check your work by differentiation.*

17. $\int 2x(x^2 - 1)^{99}\, dx$

18. $\int xe^{x^2}\, dx$

19. $\int \frac{2x^2}{\sqrt{1 - 4x^3}}\, dx$

20. $\int \frac{(\sqrt{x} + 1)^4}{2\sqrt{x}}\, dx$

21. $\int (x^2 + x)^{10}(2x + 1)\, dx$

22. $\int \frac{1}{10x - 3}\, dx$

23. $\int x^3(x^4 + 16)^6\, dx$

24. $\int \sin^{10}\theta \cos\theta\, d\theta$

25. $\int \frac{1}{\sqrt{1 - 9x^2}}\, dx$

26. $\int x^9 \sin x^{10}\, dx$

27. $\int (x^6 - 3x^2)^4(x^5 - x)\, dx$

28. $\int \frac{x}{x - 2}\, dx$ (*Hint:* Let $u = x - 2$)

29–34. Variations on the substitution method *Find the following integrals.*

29. $\int \frac{x}{\sqrt{x - 4}}\, dx$

30. $\int \frac{y^2}{(y + 1)^4}\, dy$

31. $\int \frac{x}{\sqrt[3]{x + 4}}\, dx$

32. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

33. $\int x\sqrt[3]{2x + 1}\, dx$

34. $\int (x + 1)\sqrt{3x + 2}\, dx$

35–44. Definite integrals *Use a change of variables to evaluate the following definite integrals.*

35. $\int_0^1 2x(4 - x^2)\, dx$

36. $\int_0^2 \frac{2x}{(x^2 + 1)^2}\, dx$

37. $\int_0^{\pi/2} \sin^2\theta \cos\theta\, d\theta$

38. $\int_0^{\pi/4} \frac{\sin x}{\cos^2 x}\, dx$

39. $\int_{-1}^{2} x^2 e^{x^3+1}\, dx$

40. $\int_0^4 \frac{p}{\sqrt{9 + p^2}}\, dp$

41. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^2 x}\, dx$

42. $\int_0^{\pi/4} \frac{\sin x}{\cos^3 x}\, dx$

43. $\int_{2/(5\sqrt{3})}^{2/5} \frac{dx}{x\sqrt{25x^2 - 1}}$

44. $\int_0^3 \frac{v^2 + 1}{\sqrt{v^3 + 3v + 4}}\, dv$

45–50. Integrals with $\sin^2 x$ and $\cos^2 x$ *Evaluate the following integrals.*

45. $\int_{-\pi}^{\pi} \cos^2 x\, dx$

46. $\int \sin^2 x\, dx$

47. $\int \sin^2\left(\theta + \frac{\pi}{6}\right) d\theta$

48. $\int_0^{\pi/4} \cos^2 8\theta\, d\theta$

49. $\int_{-\pi/4}^{\pi/4} \sin^2 2\theta\, d\theta$

50. $\int x\cos^2(x^2)\, dx$

Further Explorations

51. **Explain why or why not** Determine whether the following statements are true and give an explanation or counterexample. Assume that f, f', and f'' are continuous functions for all real numbers.
    a. $\int f(x)f'(x)\, dx = \frac{1}{2}(f(x))^2 + C$
    b. $\int (f(x))^n f'(x)\, dx = \frac{1}{n + 1}(f(x))^{n+1} + C,\ n \neq -1$
    c. $\int \sin 2x\, dx = 2\int \sin x\, dx$
    d. $\int (x^2 + 1)^9 dx = \frac{(x^2 + 1)^{10}}{10} + C$
    e. $\int_a^b f'(x)f''(x)\, dx = f'(b) - f'(a)$

52–64. Additional integrals *Use a change of variables to evaluate the following integrals.*

52. $\int \sec 4w \tan 4w\, dw$

53. $\int \sec^2 10x\, dx$

54. $\int (\sin^5 x + 3\sin^3 x - \sin x)\cos x\, dx$

55. $\int \frac{\csc^2 x}{\cot^3 x}\, dx$

56. $\int (x^{3/2} + 8)^5 \sqrt{x}\, dx$

57. $\int \sin x \sec^8 x\, dx$

58. $\int \frac{e^{2x}}{e^{2x} + 1}\, dx$

59. $\int_0^1 x\sqrt{1 - x^2}\, dx$

60. $\int_1^{e^2} \frac{\ln x}{x}\, dx$

61. $\int_2^3 \frac{x}{\sqrt[3]{x^2 - 1}}\, dx$

62. $\int_0^6 \frac{dx}{x^2 + 36}$

63. $\int_0^2 x^3\sqrt{16 - x^4}\, dx$

64. $\int_{\sqrt{2}}^{\sqrt{3}} (x - 1)(x^2 - 2x)^{11}\, dx$

65–68. Areas of regions *Find the area of the following regions.*

65. The region bounded by the graph of $f(x) = x\sin(x^2)$ and the x-axis between $x = 0$ and $x = \sqrt{\pi}$

66. The region bounded by the graph of $f(\theta) = \cos\theta\sin\theta$ and the θ-axis between $\theta = 0$ and $\theta = \pi/2$

67. The region bounded by the graph of $f(x) = (x - 4)^4$ and the x-axis between $x = 2$ and $x = 6$

68. The region bounded by the graph of $f(x) = \frac{x}{\sqrt{x^2 - 9}}$ and the x-axis between $x = 4$ and $x = 5$

69. **Morphing parabolas** The family of parabolas $y = (1/a) - x^2/a^3$, where $a > 0$, has the property that for $x \geq 0$, the x-intercept is $(a, 0)$ and the y-intercept is $(0, 1/a)$. Let $A(a)$ be the area of the region in the first quadrant bounded by the parabola and the x-axis. Find $A(a)$ and determine whether it is an increasing, decreasing, or constant function of a.

Applications

70. **Periodic motion** An object moves in one dimension with a velocity in m/s given by $v(t) = 8\cos(\pi t/6)$.
    a. Graph the velocity function.
    b. As will be discussed in Chapter 6, the position of the object is given by $s(t) = \int_0^t v(y)\, dy$ for $t \geq 0$. Find the position function for all $t \geq 0$.
    c. What is the period of the motion—that is, starting at any point, how long does it take the object to return to that position?

71. **Population models** The population of a culture of bacteria has a growth rate given by $p'(t) = \frac{200}{(t + 1)^r}$ bacteria per hour, for $t \geq 0$, where $r > 1$ is a real number. In Chapter 6 it will be shown that the increase in the population over the time interval $[0, t]$ is given by $\int_0^t p'(s)\, ds$. (Note that the growth rate decreases in time, reflecting competition for space and food.)
    a. Using the population model with $r = 2$, what is the increase in the population over the time interval $0 \leq t \leq 4$?
    b. Using the population model with $r = 3$, what is the increase in the population over the time interval $0 \leq t \leq 6$?
    c. Let ΔP be the increase in the population over a fixed time interval $[0, T]$. For fixed T, does ΔP increase or decrease with the parameter r? Explain.
    d. A lab technician measures an increase in the population of 350 bacteria over the 10-hr period $[0, 10]$. Estimate the value of r that best fits this data point.
    e. Looking ahead: Work with the population model using $r = 3$ (part (b)) and find the increase in population over the time interval $[0, T]$ for any $T > 0$. If the culture is allowed to grow indefinitely ($T \to \infty$), does the bacteria population increase without bound? Or does it approach a finite limit?

72. Consider the right triangle with vertices $(0, 0)$, $(0, b)$, and $(a, 0)$, where $a > 0$ and $b > 0$. Show that the average vertical distance from points on the x-axis to the hypotenuse is $b/2$ for all $a > 0$.

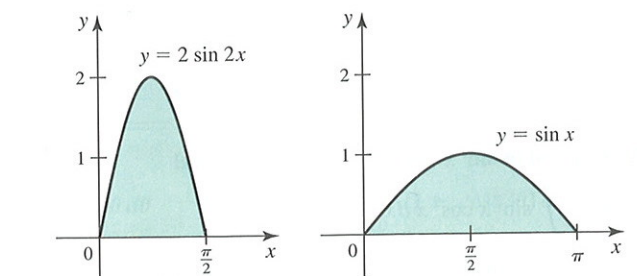
73. **Average value of sine functions** Use a graphing utility to verify that the functions $f(x) = \sin kx$ have a period of $2\pi/k$, where $k = 1, 2, 3, \ldots$. Equivalently, the first "hump" of $f(x) = \sin kx$ occurs on the interval $[0, \pi/k]$. Verify that the average value of the first hump of $f(x) = \sin kx$ is independent of k. What is the average value? (See Section 5.4 for average value.)

Additional Exercises

74. **Looking ahead** Integrals of $\tan x$ and $\cot x$
    a. Use a change of variables to show that
    $$\int \tan x\, dx = -\ln|\cos x| + C = \ln|\sec x| + C.$$
    b. Show that
    $$\int \cot x\, dx = \ln|\sin x| + C.$$

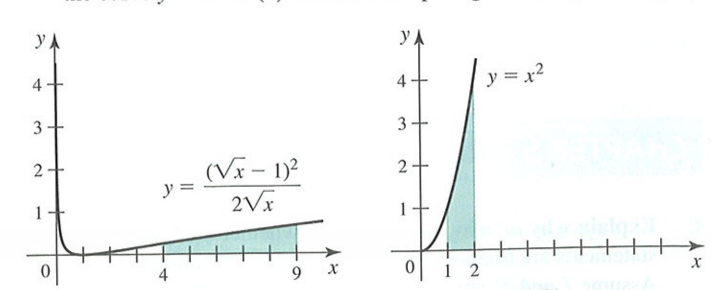
75. **Looking ahead** Integrals of $\sec x$ and $\csc x$
    a. Multiply the numerator and denominator of $\sec x$ by $\sec x + \tan x$; then use a change of variables to show that
    $$\int \sec x\, dx = \ln|\sec x + \tan x| + C.$$
    b. Show that
    $$\int \csc x\, dx = -\ln|\csc x + \cot x| + C.$$

76. **Equal areas** The area of the shaded region under the curve $y = 2\sin 2x$ in (a) equals the area of the shaded region under the curve $y = \sin x$ in (b). Explain why this is true without computing areas.



(a) (b)

77. **Equal areas** The area of the shaded region under the curve $y = \frac{(\sqrt{x} - 1)^2}{2\sqrt{x}}$ in (a) equals the area of the shaded region under the curve $y = x^2$ in (b). Without computing areas, explain why.



(a) (b)

78–82. General results *Evaluate the following integrals in which the function f is unspecified. Note $f^{(p)}$ is the pth derivative of f and f^p is the pth power of f. Assume f and its derivatives are continuous for all real numbers.*

78. $\int (5f^3(x) + 7f^2(x) + f(x))f'(x)\, dx$

79. $\int_1^2 (5f^3(x) + 7f^2(x) + f(x))f'(x)\, dx$, where $f(1) = 4$, $f(2) = 5$

80. $\displaystyle\int_0^1 f'(x)f''(x)\,dx$, where $f'(0) = 3$ and $f'(1) = 2$

81. $\displaystyle\int (f^{(p)}(x))^n f^{(p+1)}(x)\,dx$, where p is a positive integer, $n \neq -1$

82. $\displaystyle\int 2(f^2(x) + 2f(x))f(x)f'(x)\,dx$

83–85. More than one way *Occasionally, two different substitutions do the job. Use both of the given substitutions to evaluate the following integrals.*

83. $\displaystyle\int_0^1 x\sqrt{x+a}\,dx;\ a > 0 \quad (u = \sqrt{x+a} \text{ and } u = x + a)$

84. $\displaystyle\int_0^1 x\sqrt[p]{x+a}\,dx;\ a > 0 \quad (u = \sqrt[p]{x+a} \text{ and } u = x + a)$

85. $\displaystyle\int \sec^3\theta\tan\theta\,d\theta \quad (u = \cos\theta \text{ and } u = \sec\theta)$

86. **$\sin^2 ax$ and $\cos^2 ax$ integrals** Use the Substitution Rule to prove that

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a} + C \quad \text{and}$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin(2ax)}{4a} + C$$

87. **Integral of $\sin^2 x\cos^2 x$** Consider the integral

$$I = \int \sin^2 x\cos^2 x\,dx.$$

a. Find I using the identity $\sin 2x = 2\sin x\cos x$.
b. Find I using the identity $\cos^2 x = 1 - \sin^2 x$.
c. Confirm that the results in parts (a) and (b) are consistent and compare the work involved in each method.

88. **Substitution: shift** Perhaps the simplest change of variables is the shift or translation given by $u = x + c$, where c is a real number.

a. Prove that shifting a function does not change the net area under the curve, in the sense that

$$\int_a^b f(x+c)\,dx = \int_{a+c}^{b+c} f(u)\,du.$$

b. Draw a picture to illustrate this change of variables in the case that $f(x) = \sin x$, $a = 0$, $b = \pi$, $c = \pi/2$.

89. **Substitution: scaling** Another change of variables that can be interpreted geometrically is the scaling $u = cx$, where c is a real number. Prove and interpret the fact that

$$\int_a^b f(cx)\,dx = \frac{1}{c}\int_{ac}^{bc} f(u)\,du.$$

Draw a picture to illustrate this change of variables in the case that $f(x) = \sin x$, $a = 0$, $b = \pi$, $c = \frac{1}{2}$.

90–93. Multiple substitutions *Use two or more substitutions to find the following integrals.*

90. $\displaystyle\int x\sin^4(x^2)\cos(x^2)\,dx$

(*Hint:* Begin with $u = x^2$, then use $v = \sin u$.)

91. $\displaystyle\int \frac{dx}{\sqrt{1+\sqrt{1+x}}}$ $\left(\textit{Hint: } \text{Begin with } u = \sqrt{1+x}.\right)$

92. $\displaystyle\int \tan^{10}(4x)\sec^2(4x)\,dx$ (*Hint:* Begin with $u = 4x$.)

93. $\displaystyle\int_0^{\pi/2} \frac{\cos\theta\sin\theta}{\sqrt{\cos^2\theta + 16}}\,d\theta$ (*Hint:* Begin with $u = \cos\theta$.)

QUICK CHECK ANSWERS

1. $u = x^4 + 5$ **2.** With $u = x^5 + 6$, we have $du = 5x^4$, and x^4 does not appear in the integrand. **3.** New equation: $u^2 - 13u + 36 = 0$; roots: $x = \pm 2, \pm 3$ ◄

CHAPTER 5 REVIEW EXERCISES

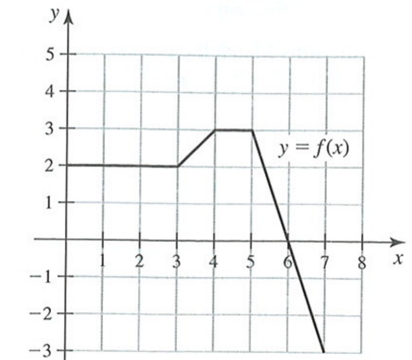
1. **Explain why or why not** Determine whether the following statements are true and give an explanation or counterexample. Assume f and f' are continuous functions for all real numbers.

a. If $A(x) = \int_a^x f(t)\,dt$ and $f(t) = 2t - 3$, then A is a quadratic function.
b. Given an area function $A(x) = \int_a^x f(t)\,dt$ and an antiderivative F of f, it follows that $A'(x) = F(x)$.
c. $\int_a^b f'(x)\,dx = f(b) - f(a)$
d. If $\int_a^b |f(x)|\,dx = 0$, then $f(x) = 0$ on $[a, b]$.
e. If the average value of f on $[a, b]$ is zero, then $f(x) = 0$ on $[a, b]$.
f. $\int_a^b (2f(x) - 3g(x))\,dx = 2\int_a^b f(x)\,dx + 3\int_b^a g(x)\,dx$
g. $\int f'(g(x))g'(x)\,dx = f(g(x)) + C$

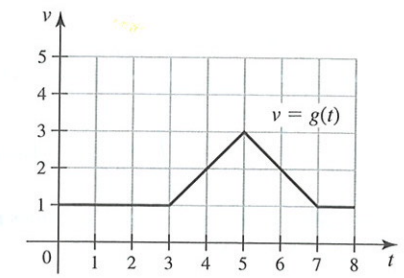
2. **Velocity to displacement** An object travels on the x-axis with a velocity given by $v(t) = 2t + 5$, for $0 \le t \le 4$.

a. How far does the object travel for $0 \le t \le 4$?
b. What is the average value of v on the interval $[0, 4]$?
c. True or false: The object would travel as far as in part (a) if it traveled at its average velocity (a constant) for $0 \le t \le 4$.

3. **Area by geometry** Use geometry to evaluate $\int_0^7 f(x)\,dx$, where the graph of f is given in the figure.



4. **Displacement by geometry** Use geometry to find the displacement of an object moving along a line for $0 \le t \le 8$, where the graph of its velocity $v = g(t)$ is given in the figure.



5. **Area by geometry** Use geometry to evaluate $\int_0^4 \sqrt{8x - x^2}\,dx$ (*Hint:* Complete the square of $8x - x^2$ first).

6. **Bagel output** The manager of a bagel bakery collects the following production rate data (in bagels per minute) at six different times during the morning. Estimate the total number of bagels produced between 6:00 and 7:30 a.m.

Time of day (a.m.)	Production rate (bagels/min)
6:00	45
6:15	60
6:30	75
6:45	60
7:00	50
7:15	40

7. **Integration by Riemann sums** Consider the integral $\int_1^4 (3x - 2)\,dx$.

a. Give the right Riemann sum for the integral with $n = 3$.
b. Use summation notation to write the right Riemann sum for an arbitrary positive integer n.
c. Evaluate the definite integral by taking the limit as $n \to \infty$ of the Riemann sum in part (b).

8. **Evaluating Riemann sums** Consider the function $f(x) = 3x + 4$ on the interval $[3, 7]$. Show that the midpoint Riemann sum with $n = 4$ gives the exact area of the region bounded by the graph.

9. **Sum to integral** Evaluate the following limit by identifying the integral that it represents:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(\frac{4k}{n}\right)^8 + 1\right]\left(\frac{4}{n}\right).$$

10. **Area function by geometry** Use geometry to find the area $A(x)$ that is bounded by the graph of $f(t) = 2t - 4$ and the t-axis between the point $(2, 0)$ and the variable point $(x, 0)$, where $x \ge 2$. Verify that $A'(x) = f(x)$.

11–26. Evaluating integrals *Evaluate the following integrals.*

11. $\displaystyle\int_{-2}^{2}(3x^4 - 2x + 1)\,dx$

12. $\displaystyle\int \cos 3x\,dx$

13. $\displaystyle\int_0^2 (x+1)^3\,dx$

14. $\displaystyle\int_0^1 (4x^{21} - 2x^{16} + 1)\,dx$

15. $\displaystyle\int (9x^8 - 7x^6)\,dx$

16. $\displaystyle\int_{-2}^{2} e^{4x+8}\,dx$

17. $\displaystyle\int_0^1 \sqrt{x}(\sqrt{x} + 1)\,dx$

18. $\displaystyle\int \frac{x^2}{x^3 + 27}\,dx$

19. $\displaystyle\int_0^1 \frac{dx}{\sqrt{4 - x^2}}$

20. $\displaystyle\int y^2(3y^3 + 1)^4\,dy$

21. $\displaystyle\int_0^3 \frac{x}{\sqrt{25 - x^2}}\,dx$

22. $\displaystyle\int x\sin x^2\cos^8 x^2\,dx$

23. $\displaystyle\int \sin^2 5\theta\,d\theta$

24. $\displaystyle\int_0^{\pi}(1 - \cos^2 3\theta)\,d\theta$

25. $\displaystyle\int \frac{x^2 + 2x - 2}{x^3 + 3x^2 - 6x}\,dx$

26. $\displaystyle\int_0^{\ln 2} \frac{e^x}{1 + e^{2x}}\,dx$

27. **Symmetry properties** Suppose that $\int_0^4 f(x)\,dx = 10$ and $\int_0^4 g(x)\,dx = 20$. Furthermore, suppose that f is an even function and g is an odd function. Evaluate the following integrals.

a. $\displaystyle\int_{-4}^{4} f(x)\,dx$ b. $\displaystyle\int_{-4}^{4} 3g(x)\,dx$ c. $\displaystyle\int_{-4}^{4}(4f(x) - 3g(x))\,dx$

28. **Properties of integrals** The figure shows the areas of regions bounded by the graph of f and the x-axis. Evaluate the following integrals.

a. $\displaystyle\int_a^c f(x)\,dx$ b. $\displaystyle\int_b^d f(x)\,dx$ c. $\displaystyle 2\int_c^b f(x)\,dx$

d. $\displaystyle 4\int_a^d f(x)\,dx$ e. $\displaystyle 3\int_a^b f(x)\,dx$ f. $\displaystyle 2\int_b^d f(x)\,dx$